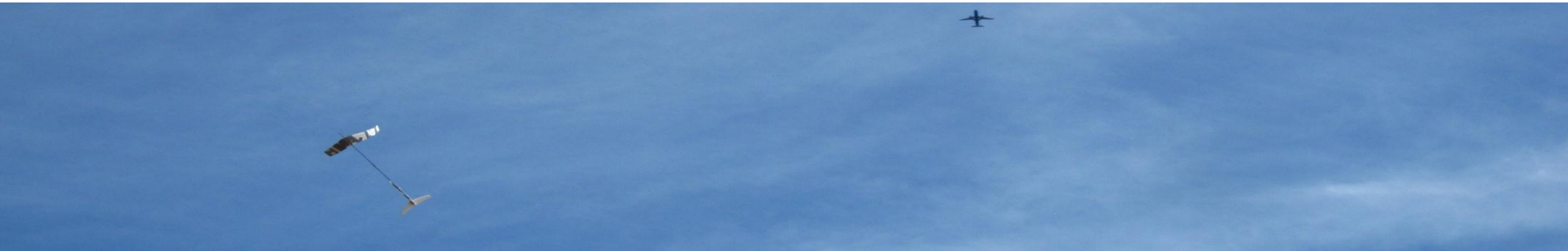
STABILITY OF PLATFORMS FOR AIRBORNE WIND TURBINES

Sara Smoot

Advised by Ilan Kroo

Stanford University



Motivation

Initial attempts at flying a tethered glider

Introduction

Goal: Develop a design analysis tool to compare the dynamics of tethered air platforms.

- Use existing aerodynamic analysis tools
- Simple model with minimal tether dynamics
  - Low altitude, ie. short tether
  - Thin, light tether
- Evaluate stability
- Investigate modal dynamics
- Facilitate intuition for dynamics
- Accommodate rapid design iteration

How will design elements effect stability?

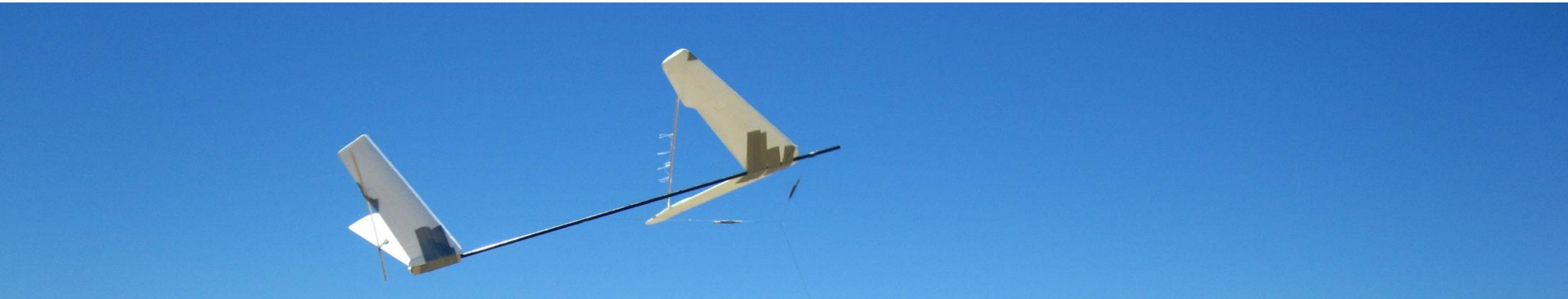
Approach

Adapt linear airplane equations of motion to tethered wings

- Develop a model that includes tether dynamics
- Choose a set of states that describe the system
- Linearize about a convenient operational point
- Check for decoupling
- Extract information about linearized modes

Compare results to tethered gliders

- Qualitative comparison of linearized modes
- Qualitative stability comparison



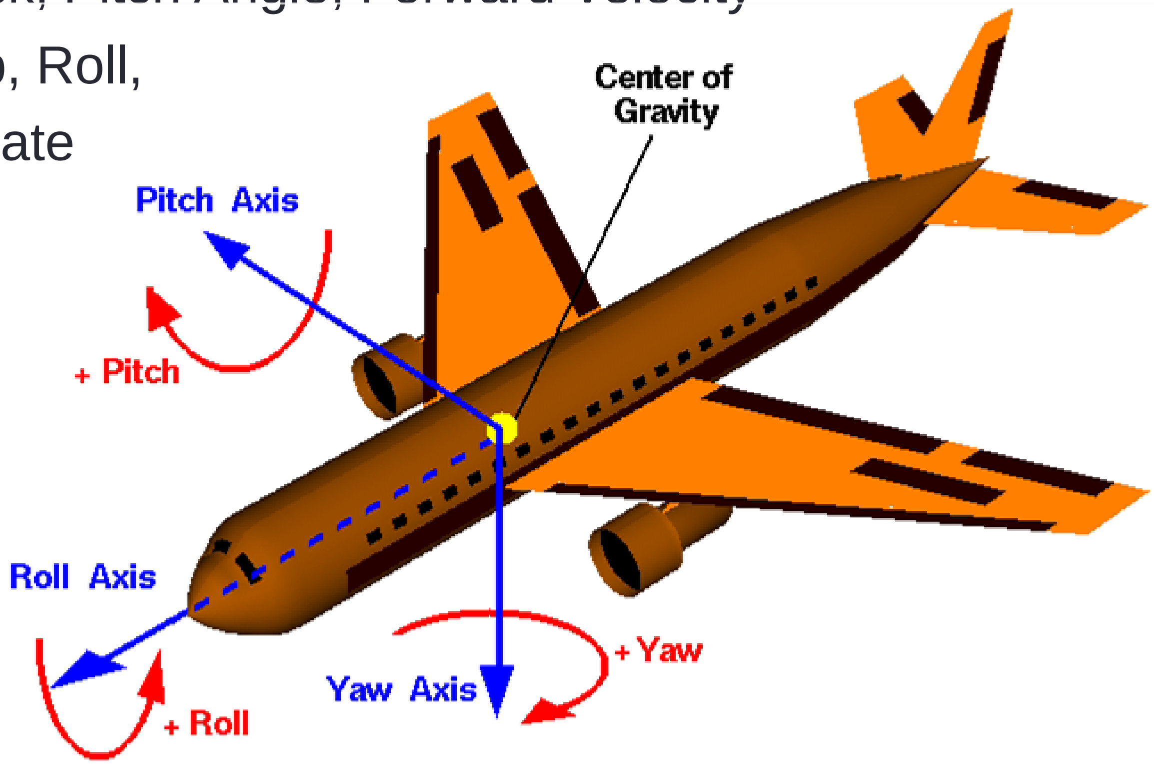
Airplane Summary

Aircraft dynamics are linearized around operational points

- Reduce the aircraft dynamics to 8 dimensionless states:
  - $\{\alpha, \theta, u\}$ Angle of attack, Pitch Angle, Forward Velocity
  - $\{\beta, \phi, \dot{\phi}, \varphi, \dot{\varphi}\}$ Side Slip, Roll, Roll Rate, Yaw, Yaw Rate

States decouple for steady level flight

- Longitude $\{\alpha, \theta, u\}$
- Lateral $\{\beta, \phi, \dot{\phi}, \varphi, \dot{\varphi}\}$



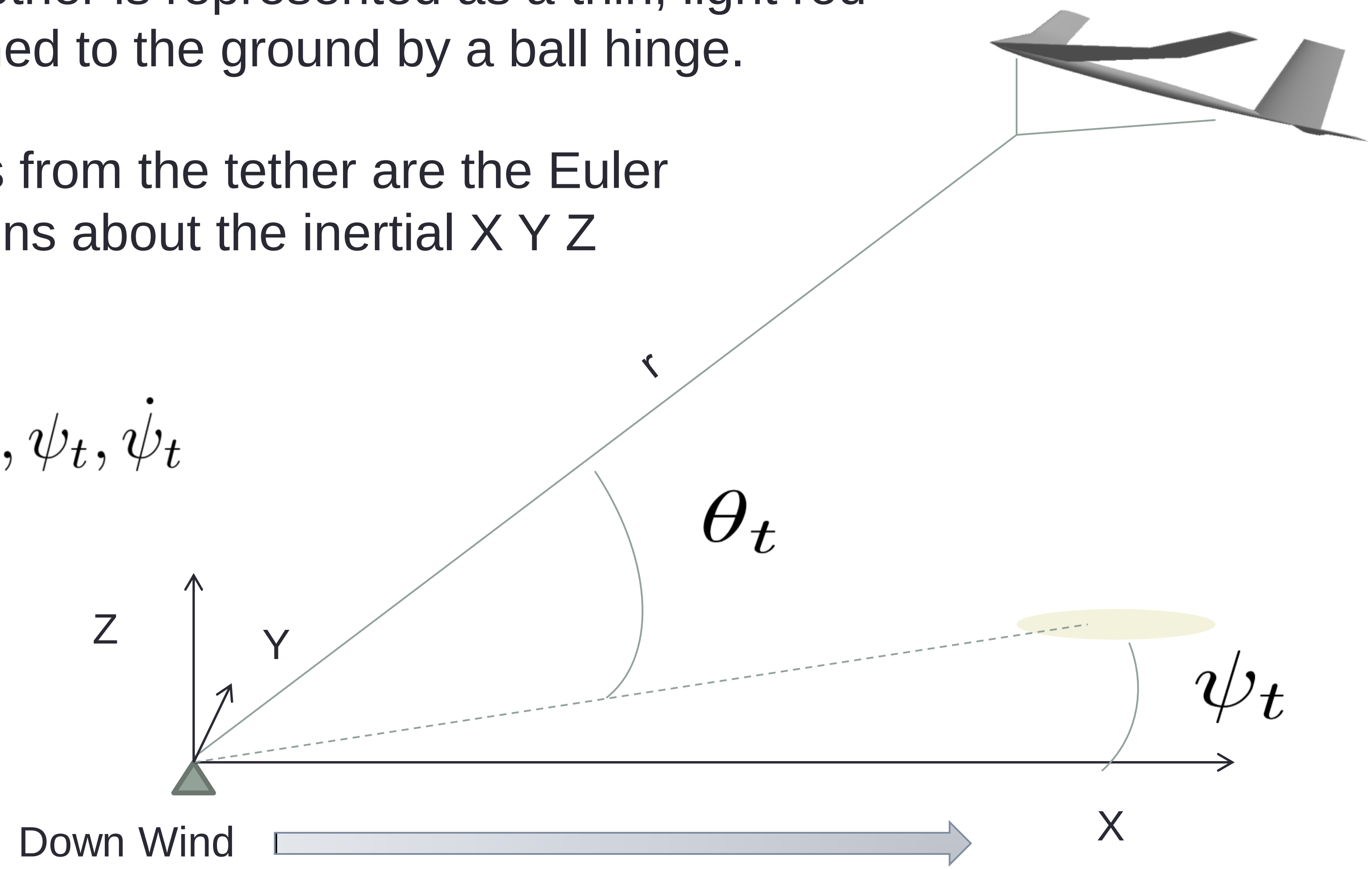
http://www.grc.nasa.gov/WWW/K-12/airplane/rotations.html

Simplified Model: Tether

The tether is represented as a thin, light rod attached to the ground by a ball hinge.

States from the tether are the Euler rotations about the inertial X Y Z frame.

$$\theta_t, \dot{\theta}_t, \psi_t, \dot{\psi}_t$$



Simple Model: Wing + Bridle

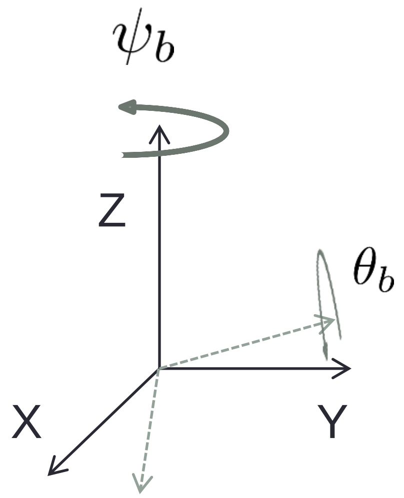
The bridled wing is connected to the tether by another ball hinge and is treated as a rigid body.

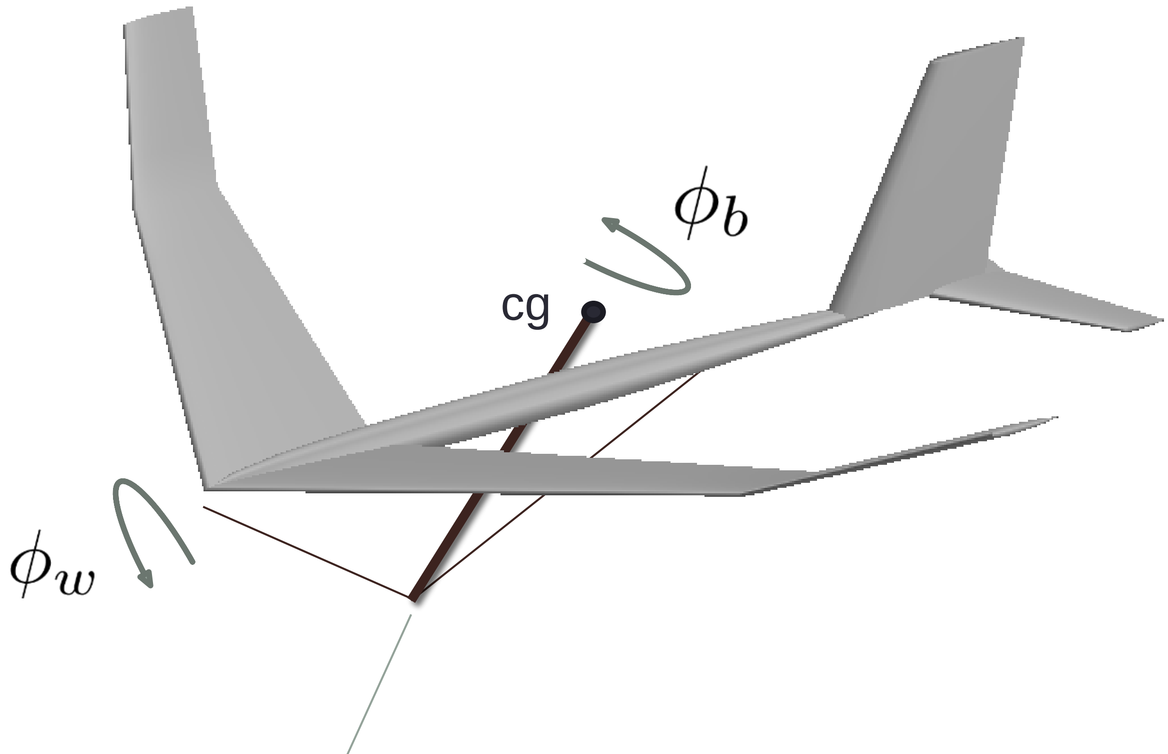
States are the Euler rotations measured along the vector from the bridal joint to the center of mass.

An additional rotation about the standard body x axis is included.

Depending on the type of bridle the wing can roll independently.

$$\theta_b, \dot{\theta}_b, \psi_b, \dot{\psi}_b, \phi_b, \dot{\phi}_b, \phi_w, \dot{\phi}_w$$





Equations of Motion

$$m\ddot{x}_k = F_g + F_w + F_a + F_t$$
$$I\dot{\omega}_k + \omega_k \times I\omega_k = M_w$$
$$\dot{H} - \vec{b} \times m\ddot{x}_b = \vec{b} \times (F_g + F_w) + M_w$$

M_w	Aerodynamic Moments from Wing
ω_t	Tether Rotational Rates
ω_b	Bridle Rotational Rates
ω_k	Wing Rotational Rates
F_g	Gravitational Force on Wing
F_w	Aerodynamic Forces from Wing
F_a	Tether Drag
F_t	Tether Tension

$$x_b = \begin{bmatrix} r & 0 & 0 \end{bmatrix}$$
$$\dot{x}_b = \omega_t \times x_b$$
$$\ddot{x}_b = \dot{\omega}_t \times x_b + \omega_t \times \omega_t \times x_b$$

$$x_k = \begin{bmatrix} b & 0 & 0 \end{bmatrix}$$
$$\dot{x}_k = T_{BK}\dot{x}_b + \omega_b \times x_k$$
$$\ddot{x}_k = T_{BK}\ddot{x}_b + \dot{\omega}_b \times x_k + \omega_b \times \omega_b \times x_k$$

Linearized States

Linearize around the static downwind state to obtain the linear system:

$$\dot{x} = Ax$$

$$x = \begin{bmatrix} \theta_t & \dot{\theta}_t & \psi_t & \dot{\psi}_t & \theta_b & \dot{\theta}_b & \psi_b & \dot{\psi}_b & \phi_b & \dot{\phi}_b & \phi_w & \dot{\phi}_w \end{bmatrix}^T$$

The states can be ordered such that matrix A is block diagonal

$$\begin{bmatrix} \dot{x}_{long} \\ \dot{x}_{lat} \end{bmatrix} = \begin{bmatrix} A_{long} & 0 \\ 0 & A_{lat} \end{bmatrix} \begin{bmatrix} x_{long} \\ x_{lat} \end{bmatrix}$$

Lateral States $$x_{lat} = \begin{bmatrix} \psi_t & \dot{\psi}_t & \psi_b & \dot{\psi}_b & \phi_b & \dot{\phi}_b & \phi_w & \dot{\phi}_w \end{bmatrix}^T$$

Longitudinal States $$x_{long} = \begin{bmatrix} \theta_t & \dot{\theta}_t & \theta_b & \dot{\theta}_b \end{bmatrix}^T$$

Linearized Longitudinal Modes

		Tether Pendulum Mode	Bridle Pendulum Mode
Example Eigen Values		-0.4142 + 1.2527i -0.4142 - 1.2527i	-8.7413 + 1.9330i -8.7413 - 1.9330i
Associated Eigen Vectors	θ_t $\dot{\theta}_t$ θ_b $\dot{\theta}_b$	-0.1879 - 0.5683i 0.7897 0.0738 - 0.0342i 0.0123 + 0.1066i	-0.0001 - 0.0003i 0.0017 + 0.0026i 0.1084 + 0.0240i -0.9938

Approximate Tether Pendulum Frequency

$$\sqrt{\frac{F_a + F_w + F_g}{rm}}$$

$$A_{long} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ C_{\theta_t\theta_t} & C_{\theta_t\dot{\theta}_t} & C_{\theta_t\theta_b} & C_{\theta_t\dot{\theta}_b} \\ 0 & 0 & 0 & 1 \\ C_{\theta_b\theta_t} & C_{\theta_b\dot{\theta}_t} & C_{\theta_b\theta_b} & C_{\theta_b\dot{\theta}_b} \end{bmatrix}$$

Lateral Linearized Modes

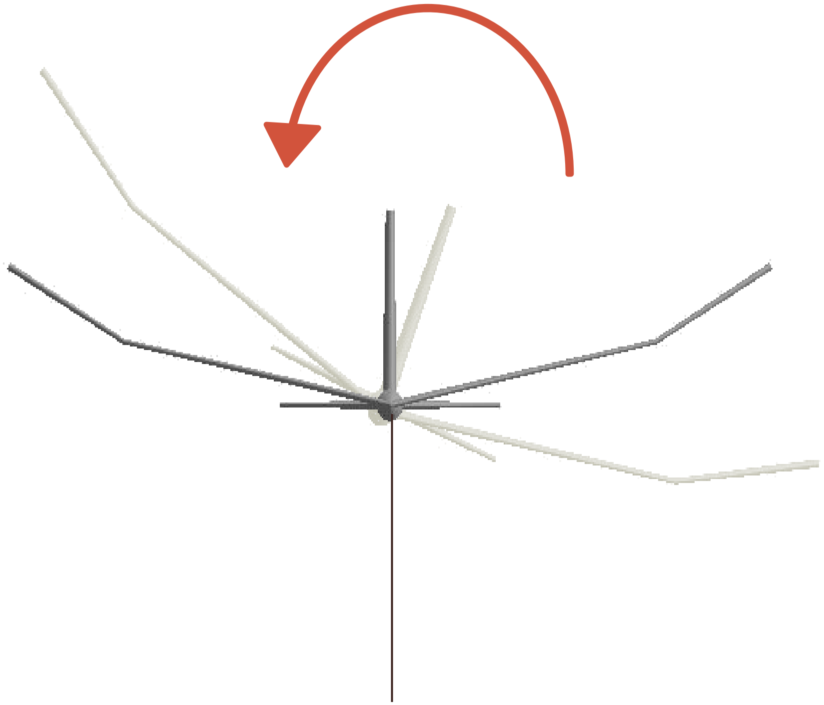
$$A_{lat} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\psi_t \psi_t} & C_{\psi_t \dot{\psi}_t} & C_{\psi_t \psi_b} & C_{\psi_t \dot{\psi}_b} & C_{\psi_t \phi_b} & C_{\psi_t \dot{\phi}_b} & C_{\psi_t \phi_w} & C_{\psi_t \dot{\phi}_w} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ C_{\psi_b \psi_t} & C_{\psi_b \dot{\psi}_t} & C_{\psi_b \psi_b} & C_{\psi_b \dot{\psi}_b} & C_{\psi_b \phi_b} & C_{\psi_b \dot{\phi}_b} & C_{\psi_b \phi_w} & C_{\psi_b \dot{\phi}_w} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ C_{\phi_b \psi_t} & C_{\phi_b \dot{\psi}_t} & C_{\phi_b \psi_b} & C_{\phi_b \dot{\psi}_b} & C_{\phi_b \phi_b} & C_{\phi_b \dot{\phi}_b} & C_{\phi_b \phi_w} & C_{\phi_b \dot{\phi}_w} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ C_{\phi_w \psi_t} & C_{\phi_w \dot{\psi}_t} & C_{\phi_w \psi_b} & C_{\phi_w \dot{\psi}_b} & C_{\phi_w \phi_b} & C_{\phi_w \dot{\phi}_b} & C_{\phi_w \phi_w} & C_{\phi_w \dot{\phi}_w} \end{bmatrix}$$

Five Lateral Modes

- Roll Subsistence
- Modified Dutch Roll
- Yaw Subsistence
- Lateral Tether Pendulum
- Coupled Roll

Lateral Modes

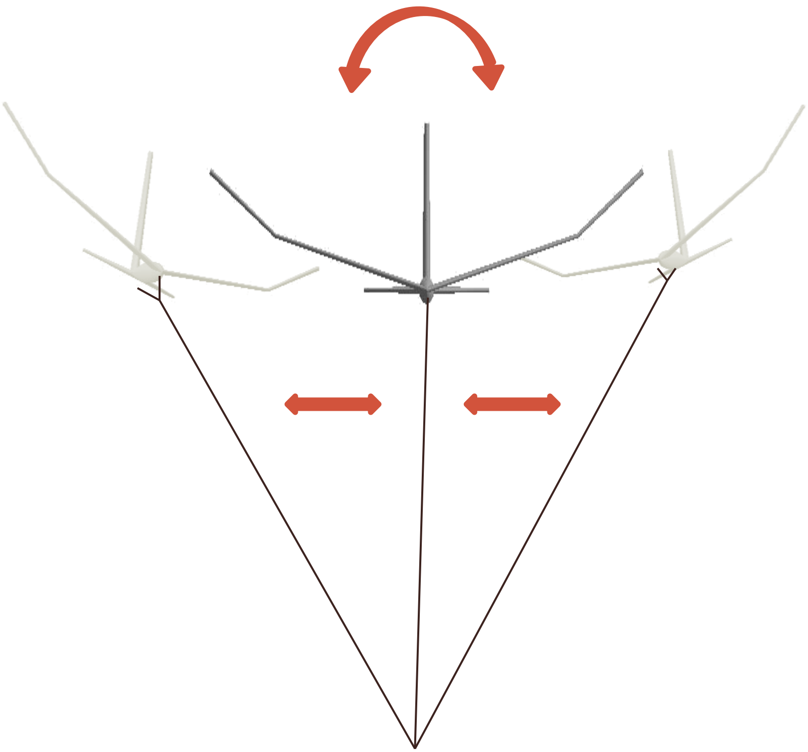
		Roll Subsistence
Example Eigen Values		*-467.73*
Associated Eigen Vectors	ψ_t	*-0.0002*
	$\dot{\psi}_t$	*0.0958*
	ψ_b	*0.0002*
	$\dot{\psi}_b$	*-0.0816*
	ϕ_b	*-0.0000*
	$\dot{\phi}_b$	*0.0009*
	ϕ_w	*-0.0021*
	$\dot{\phi}_w$	*0.9920*



- Roll subsistence: equivalent to airplane roll subsistence
  - Absent when roll is bridled

Lateral Modes

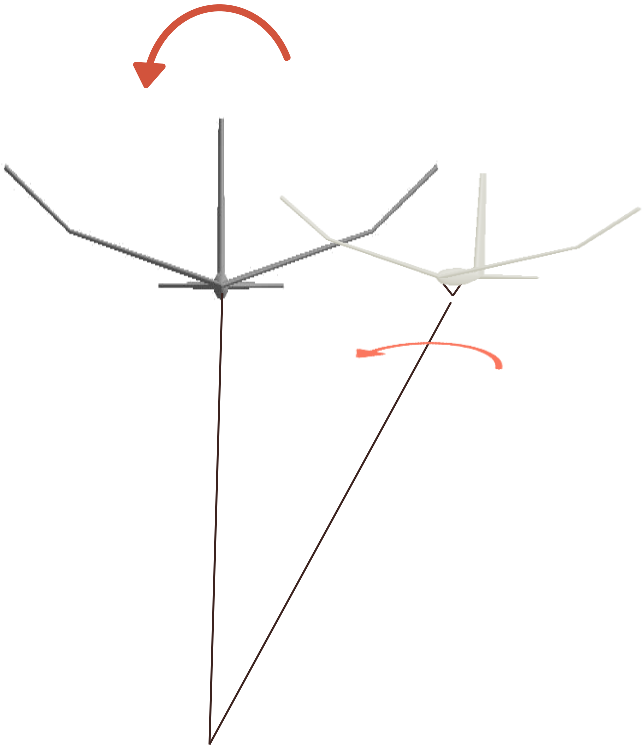
		Modified Dutch Roll
Example Eigen Values		-9.57 + 6.43i -9.57 - 6.43i
Associated Eigen Vectors	ψ_t	0.0480 + 0.0192i
	$\dot{\psi}_t$	-0.5830 + 0.1252i
	ψ_b	-0.0492 - 0.0331i
	$\dot{\psi}_b$	0.6838
	ϕ_b	0.0006 + 0.0004i
	$\dot{\phi}_b$	-0.0079 + 0.0000i
	ϕ_w	-0.0285 - 0.0215i
	$\dot{\phi}_w$	0.4110 + 0.0220i



- Modified Dutch Roll: equivalent to airplane Dutch Roll

Lateral Modes

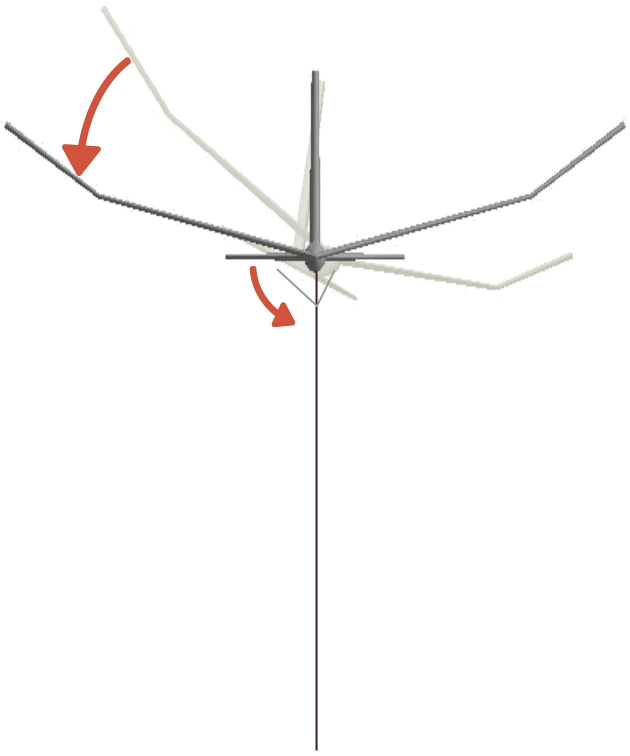
		Yaw Subsistence
Example Eigen Values		-10.24
Associated Eigen Vectors	ψ_t	-0.0483
	$\dot{\psi}_t$	0.4944
	ψ_b	0.0797
	$\dot{\psi}_b$	-0.8167
	ϕ_b	-0.0009
	$\dot{\phi}_b$	0.0092
	ϕ_w	0.0275
	$\dot{\phi}_w$	-0.2813



- Yaw Subsistence
  - Return to downwind position

Lateral Modes

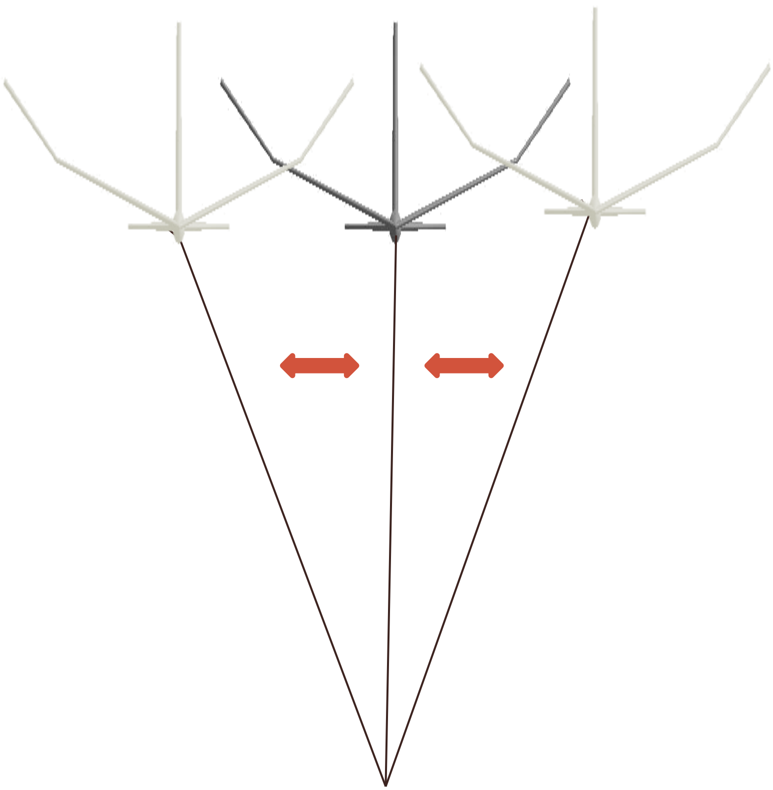
		Coupled Roll
Example Eigen Values		-0.0001 + 0.0086i -0.0001 - 0.0086i
Associated Eigen Vectors	ψ_t	-0.0159 + 0.1096i
	$\dot{\psi}_t$	-0.0939 - 0.0148i
	ψ_b	-0.0341 + 0.0130i
	$\dot{\psi}_b$	-0.0108 - 0.0294i
	ϕ_b	0.5294 - 0.0110i
	$\dot{\phi}_b$	0.0040 + 0.4544i
	ϕ_w	0.5309
	$\dot{\phi}_w$	-0.0055 + 0.4556i



- Coupled Roll
  - All the derivatives effect this one
  - Trade off with Pendulum mode

Lateral Modes

		Lateral Tether Pendulum
Example Eigen Values		-.43 + .77i -.43 - .77i
Associated Eigen Vectors	ψ_t $\dot{\psi}_t$ ψ_b $\dot{\psi}_b$ ϕ_b $\dot{\phi}_b$ ϕ_w $\dot{\phi}_w$	0.7203 -0.3062 + 0.5566i 0.0565 - 0.1600i 0.0997 + 0.1117i -0.0191 - 0.0795i 0.0695 + 0.0190i 0.0538 - 0.0728i 0.0334 + 0.0725i



- Lateral Tether Pendulum
  - Lateral swinging of tether ball joint

Flight Demonstration

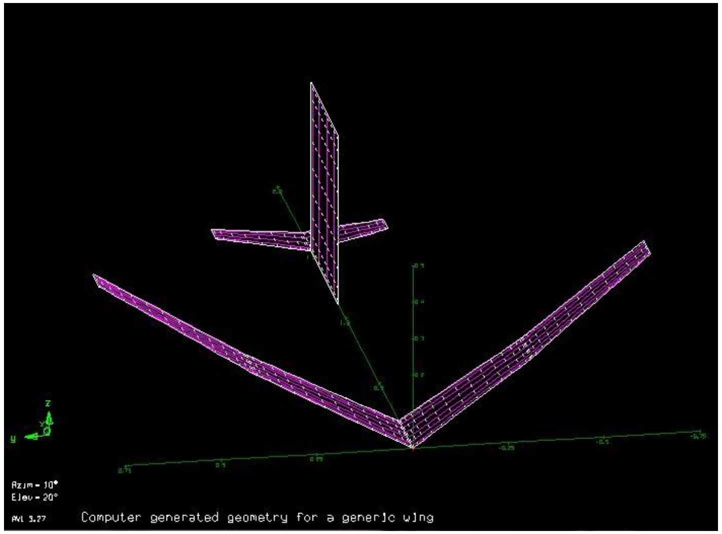
Dominate Modes:

Lateral Pendulum
Modified Dutch Roll

Flight Tests Qualitative Comparison

Built 23 different wings and varied:

- Tail length and Size
- Sweep
- Dihedral and multiple dihedral joints
- Span
- Horizontal Tail



Simulation

- Aerodynamic forces and moments computed using AVL
- EQM integrated using MATLAB ode45
    - Trim was found by running an interpolated quasi linear aero-model

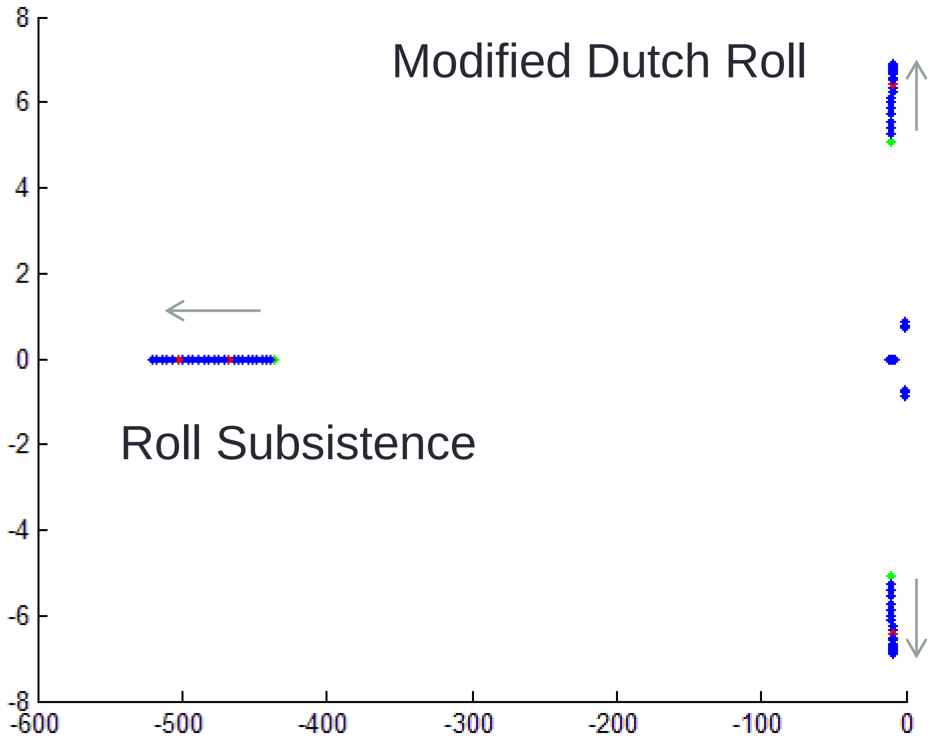
Qualitative Stability Check

Stable **Marginally Stable** **Unstable**

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Tested																							
Model																							

Design Trends

Locus of Lateral Modes with Change in Dihedral Angle



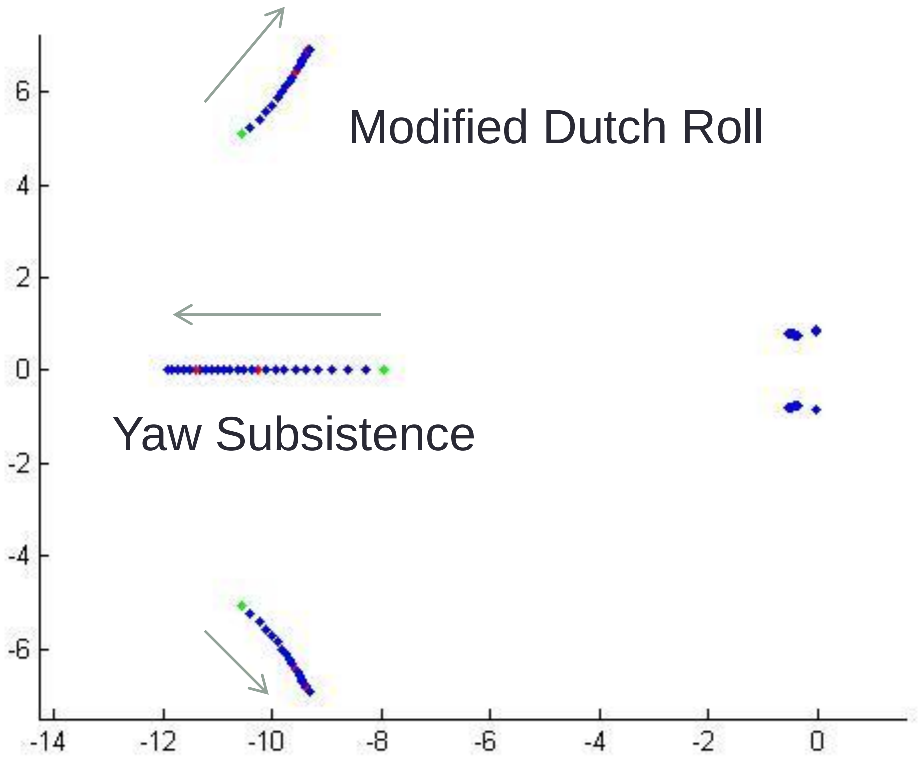
- Built Designs
- No Dihedral

Dihedral Decreases Roll Subsistence Decay Time

Dihedral Decreases Dutch Roll Damping

Design Trends

Locus of Lateral Modes with Change in Dihedral Angle



- Built Designs
- No Dihedral

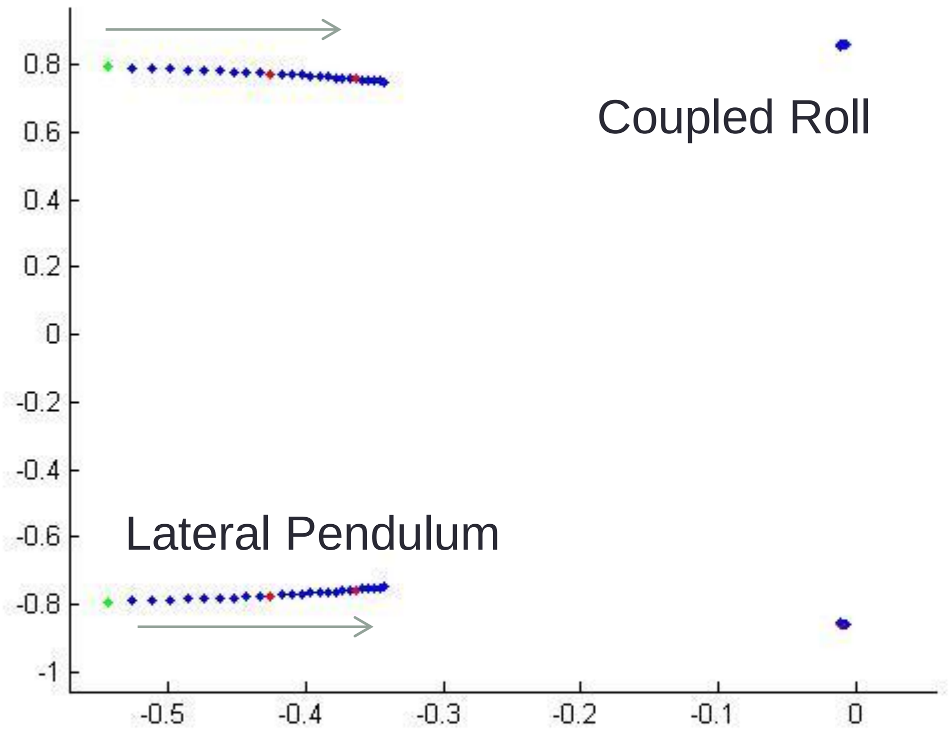
Dihedral Decreases Roll Subsistence Decay Time

Dihedral Decreases Damping Dutch Roll

Dihedral Decreases Yaw Subsistence Decay Time

Design Trends

Locus of Lateral Modes with Change in Dihedral Angle



- Built Designs
- No Dihedral

Dihedral Decreases Roll Subsistence Decay Time

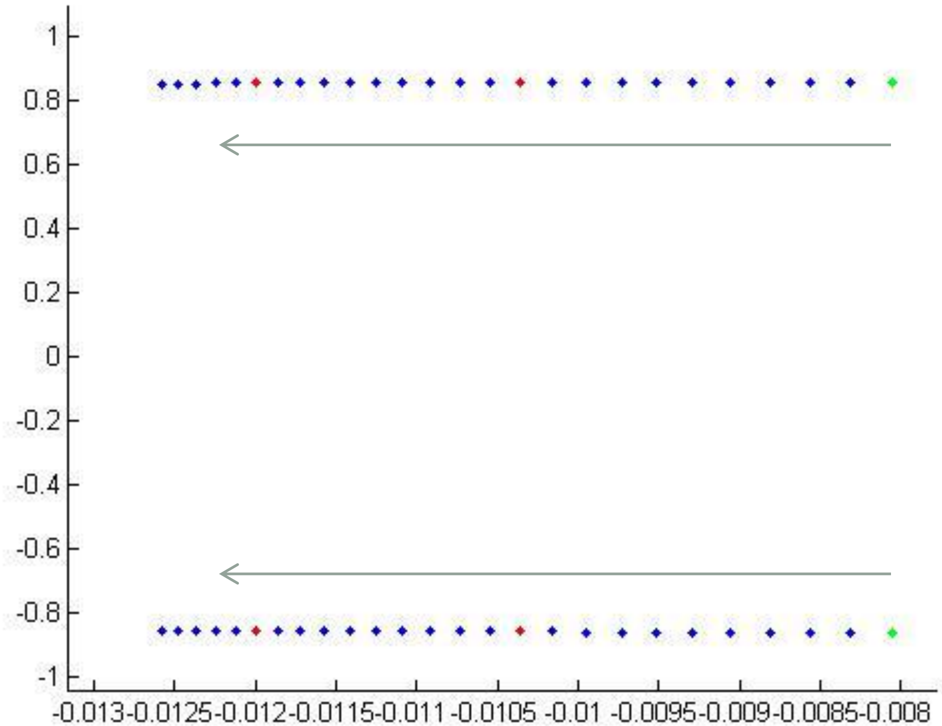
Dihedral Decreases Damping Dutch Roll

Dihedral Decreases Yaw Subsistence Decay Time

Dihedral Decreases Lateral Pendulum Damping

Design Trends

Locus of Lateral Modes with Change in Dihedral Angle



Coupled Roll Mode

- Built Designs
- No Dihedral

Dihedral Decreases Roll Subsistence Decay Time

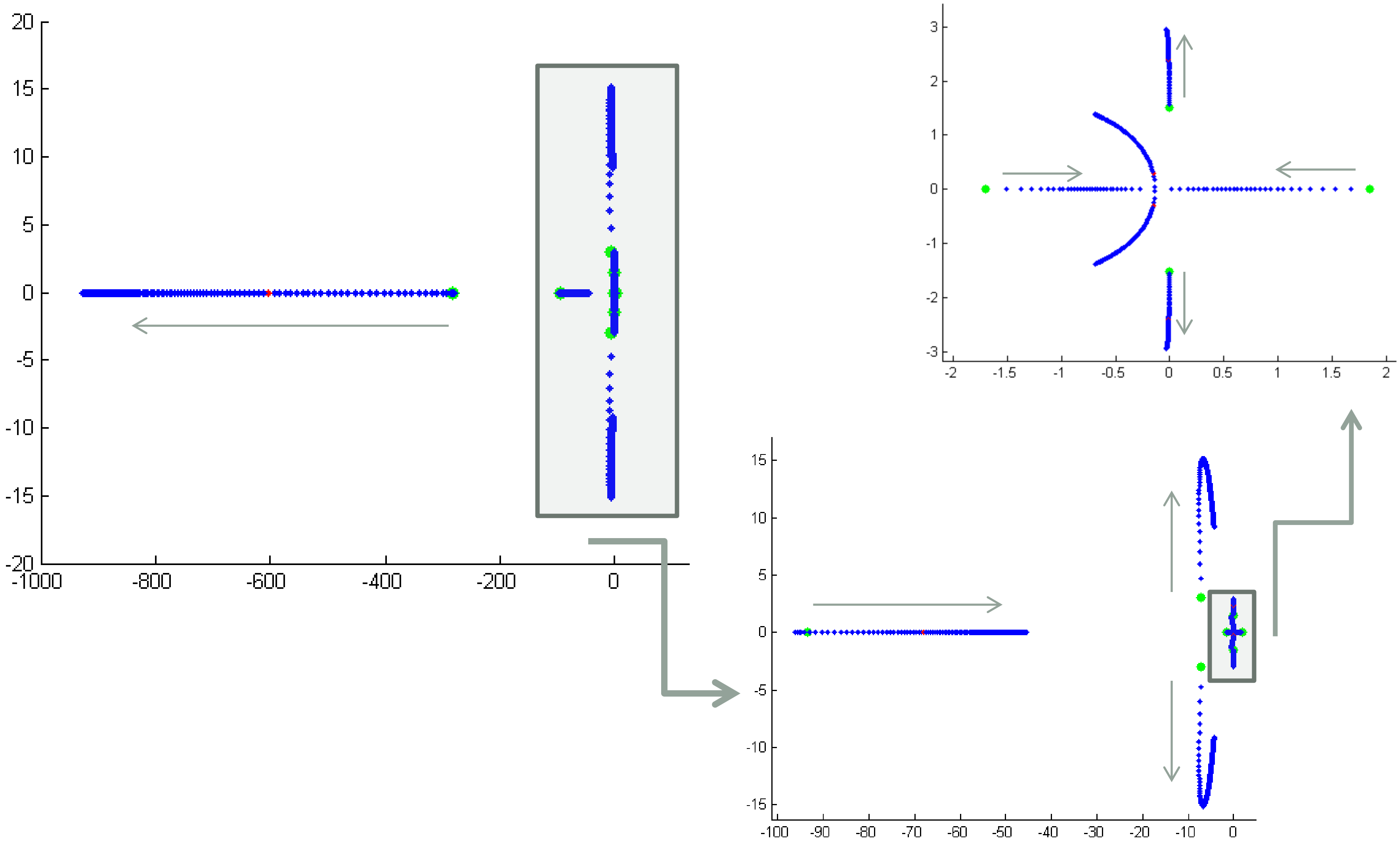
Dihedral Decreases Damping Dutch Roll

Dihedral Decreases Yaw Subsistence Decay Time

Dihedral Decreases Lateral Pendulum Damping

Dihedral Increases Coupled Roll Stability

Design Trends: Aspect Ratio



Conclusions

- A simple model of airborne tethered wings gives useful information
- Linearization can be an effective tool for:
  - Low Fidelity Design Optimization
  - Demonstrating effect of design elements on dynamics modes
  - Evaluating stability
- Next Steps
  - Develop a linearized tether model for high altitude systems and thick tethers
  - Analyze the usefulness of the model for control design